

A principal positioning system for the Earth *

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Abstract

The project SYPOR wishes to use the global navigation satellite system GALILEO as an autonomous relativistic positioning system for the Earth. Motivations and a sketch of the basic concepts underlying the project are presented. For non geodetic (perturbed) satellites, a two-dimensional example describes how the dynamics of the constellation of satellites and that of the users may be deduced from the knowledge of the dynamics of only one of the satellites during a partial interval.

1 Introduction

The current conception of the global navigation satellite systems (GNSS), like GPS, is based on a Newtonian model corrected numerically of some "relativistic effects". The *direct* relativistic theory suggests not only amelioration in accuracy, but also new functions for such GNSS systems.

The project *SYPOR* (*Système de Positionnement Relativiste*) proposes the ideas and instruments needed to carry out these new possibilities. Particularly it aims to endow the constellation of satellites of GALILEO of the necessary elements to constitute, by itself, a *primary, autonomous positioning system* for the Earth and its neighbors. The word "autonomous" refers here to the capability of the constellation to provide complete relativistic metric information, i.e. to describe the *kinematics* and the *dynamics* both, of the constellation itself and of the users (possibility of gravimetry).

For this goal, the project *SYPOR* envisages, for the first time in physics and astronomy, to construct in the neighbors of the Earth a relativity-compatible physical coordinate system (relativistic positioning system).

In this paper we sketch the general lines of the project (Section 2), as well as the underlying physical concepts, namely that of *relativistic positioning systems* (Section 3) and the way to make such systems *autonomous*, which is illustrated in the two-dimensional case (Section 4).

Some of the ideas and results on this subject are the fruit of a long and friendly collaboration with Lluís Bel (Univ. Pais Vasco, Bilbao, Spain), Joan Josep Ferrando and Juan Antonio Morales (Univ. Valencia, Burjassot, Spain), and Albert Tarantola (Inst. Physique du Globe, Paris, France).

2 Sketch of the Project

For most of the needs of geodesy and positioning, the Earth may be considered as a *Newtonian system*, for which classical mechanics is enough to explain its essential properties. But a constellation of satellites around the Earth, endowed with clocks exchanging their proper times, is a *relativistic system* in its own right (mainly due to Doppler and gravitational potential "relativistic effects"). Consequently, the natural conceptual frame to study GNSS is relativity theory.

At present, the GNSS involve the Earth and the constellation of satellites as a sole, coupled system. They start from a terrestrial, non relativistic coordinate system (Coll, 2001) and use the satellites of the constellation as moving beacons to indicate to the users their position with respect to this system.

The project SYPOR offers this result in two steps, that have different levels of conceptual precision and practical accuracy:

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1. At the first level, SYPOR proposes the concepts and means to use the sole constellation of satellite-borne clocks as the most accurate, primary, autonomous, relativistically valid, positioning system for the neighbors of the Earth. At this level, any user may know its coordinates with respect to the satellites, its dynamical state (acceleration, rotation), the exact internal configuration of the constellation, and their situation with respect to the ICRS (i.e. all what an user may hope to obtain from a primary system), and any two users may know their relative position, distance and relative orientation.

Such a positioning system is a non usual one, with light-like (rapidly variable) coordinate surfaces, but any conventional system may be defined with respect to it. It is to be noted that, at this level, no synchronization of clocks is at all necessary.

2. At the second level, the usual data of the control segment on the trajectories of the satellites, are "read" as the data defining the coordinate change to (secondary, non relativistic) terrestrial coordinates (WGS 84 or ITRF classical eference systems).

At this level, any user may know its position (with terrestrial precision) with respect to the Earth, as with the current GPS conception.

The possibility appears for a space agency to concentrate its interest in the first level, the autonomous positioning system, and delegate to global and local Earth agencies the control of the terrestrial coordinates.

To realize these performances, every satellite must be endowed with the following kinematic devices:

- a device, on every satellite, allowing to exchange proper times with its neighbors (internal control of the parts of the system) (Hammesfahr, 1999),
- a device, on four at least of the satellites, pointing to the ICRS (International Celestial Reference System) in order to define virtual local charts "at rest" with respect to the ICRS (external control of the system as a whole),
- a device, on every satellite, broadcasting over the Earth, beside its proper time, those of their neighbors (strong integrity: control by the users segment).

3 Positioning Systems

Relativity theory may be used:

- * as a wise algorithm to sprinkle Newtonian expressions with terms corresponding to the "relativistic effects" necessary for the obtention of the correct numerical values, or
- * as the adequate starting frame to rise and to approach the physical situations with the most recent concepts and developments on the space-time.

The first use is undoubtedly correct in some particular scenarios, as may be in approximate numerical computations or for the abstract comparison of the equations of the two theories. The analysis shows that in fact this first use may be correct in the situations in which the physical determination of the coordinates either does not matter, or may be numerically identified to their Newtonian geometric determinations. But it is obviously useless to take advantage of the progress and specific developments of relativity theory in its proper domain, as it is the case in advanced GNSS.

The basic arena of relativity theory is its space-time. Relativistic space-time differs from Newtonian space-time in the following essential point: "the space" and "the present" are not now "physical objects", but inessential local arbitrary conventions. The three-dimensional Newtonian space has as much physical reality as have the Ptolemaic cristal spheres, and "past", "present" and "futur" are not exhaustive complementary parts of the space-time. Consequently, "objects" in their usual sense, like galaxies, stars, planets, mountains do not exist in the relativistic space-time. What one can find in it are rather the "absolute invariants" that they generate, that is to say, their "histories". It is with the histories of the satellites that a relativistic theory of GNSS must be constructed.

A *locationing system* is a detailed description for the physical construction of a coordinate system. To construct a coordinate system is either to construct its coordinate lines, or to construct its coordinate (hyper-)surfaces. But the choice of one or the other, and the protocol of their physical construction, give rise to very different physical properties (Coll, 2001).

Thus, when the protocol of their physical construction allows a particular observer (generally situated at the origin) to attach to every point of his neighbor a set of coordinates, one has what is currently called

a *reference system*. But if this protocol allows every point of a neighbor to know its proper coordinates, then one has a *positioning system*.

In Newtonian physics, these functions are exchangeable, and perhaps this is why they are frequently mistaken. But in relativistic physics these two functions have very different physical properties:

- * they are always *incompatible* for a sole coordinate system. So that the "reference" or "positioning" character of a locationing system must be previously chosen,
- * it is always *impossible* to construct a positioning system starting from a reference system,
- * it is always *possible* (and very easily) to construct a reference system starting from a positioning one.

As a consequence, the *first* element to be conceived in a relativistic GNSS must imperatively be its positioning system, and not its terrestrial reference system (WG 84 or IERS) as it is currently the case.

Remember (Coll, 2001) that relativistic positioning systems are *generic*, *free*, and *immediate*, three very important physical properties that no other locationing system may simultaneously offer. Thus, for example, Cartesian coordinates are not generic, the standard synchronization (two-way signals) usual in the construction of reference systems does not give rise to immediate systems; harmonic conditions are generic, but not free, etc.

A general analysis of rigorous mathematical results, physical possibilities and present technical developments leads to the important epistemic result that, among all the relativistic locationing systems, the set of relativistic positioning systems exists but constitute a very little class. And the simplest representative of this class is the one formed by electromagnetic signals broadcasting the proper time τ_i of four independent clocks S_i ($i = 1, \dots, 4$).

From now on, we suppose these clocks carried by (not necessarily geodetic) satellites.

In the space-time, the above wave fronts signals, parameterized by the proper time of the clocks, draw four families of physical hypersurfaces moving at the velocity of light, realizing a *covariantly null coordinate system* $\{\tau_i\}$.

Coordinate systems of this class are very unusual. Very different from the current relativistic ones, and still more from the Newtonian ones, they have been studied by a very restricted number of specialists in relativity. For an almost exhaustive bibliography, see (Blagojević et al., 2001).

In such covariantly null coordinate systems, instead of $\eta_{\alpha\beta} = \text{diag}\{1, -1, -1, -1\}$, the Minkowski metric of the space-time adopts the complementary form:

$$\eta_{\alpha\beta} = \begin{pmatrix} 0 & f & g & h \\ f & 0 & \ell & m \\ g & \ell & 0 & n \\ h & m & n & 0 \end{pmatrix} \quad (1)$$

where f, g, h, ℓ, m, n , are strictly positive functions of the coordinates τ_i .

We see that in such coordinates there is no time-like asymmetry, the four coordinate surfaces playing exactly the same role. The nullity of all the diagonal terms in the above real expression seems to have erroneously suggested in the past that such coordinate systems would be "somewhere degenerate"; this uncorrect intuitive feeling is perhaps the cause of the absence of studies on them and of their slow re-discovery by some authors.

The four proper times $\{\tau_i\}$ read at a space-time event by a receptor constitute its (covariantly null) coordinates with respect to the four satellites. But such a system can not be considered as *primary* (with respect to the space-time structure) if we have not sufficiently information to relate to it any other coordinate system (Cartesian, harmonic, etc). And for this task, we need to know the (dynamical) space-time trajectories of the satellites. In principle, there are many ways to do that, one simple one being to force satellites to follow prescribed trajectories, for example geodesics. But the most complete one is that in which these information is generated and broadcasted at every instant by the system of satellites itself, *whatever* be their trajectories. When this happens, we call the primary system *autonomous*.

4 Autonomous Positioning Systems

How to make autonomous such a system of embarked clocks, arbitrarily synchronized, broadcasting their proper times? The answer is very simple: broadcasting, not only their proper time, but also the proper time of their neighboring satellite's clocks.

In other words: let τ_{ij} , $i \neq j$, be the proper time of the satellite's clock j received by the satellite i at its proper time instant τ_i . Then, the broadcasting of the data $\{\tau_i, \tau_{ij}\}$ allow to make autonomous the covariantly null space-time positioning system $\{\tau_i\}$.

Observe that the sixteen data $\{\tau_i, \tau_{ij}\}$ received by an observer contains, of course, the coordinates $\{\tau_i\}$ ($i = 1, \dots, 4$), of this observer but also the coordinates $\{\tau_i, \tau_{ij}\}$ of *every* satellite i in the totally covariant null coordinate system that the four satellites are generating.

In a four-dimensional Cartesian grid of axis τ_i ($i = 1, \dots, 4$), the data $\{\tau_i, \tau_{ij}\}$ received by an user represent at every instant the proper position of the user as well as the positions of the satellites. During an arbitrary interval, these data allow the user to draw in this grid its proper space-time trajectory as well as the trajectory of the four satellites. Of course, two different users will found, in their corresponding grids, that their personal trajectories are different, but in the covering domain of proper times, the trajectories of the four satellites will be necessarily the same.

Such grids are diffeomorphic to the corresponding domain of the space-time, but they are not *isometric*. The precise deformation that relates them is nothing but the *metric tensor* $g_{ij}(\tau_\ell)$ of this domain, which gives the dynamical properties of the trajectories (their inertial and/or gravitational characteristics).

When the grid trajectories of the satellites are particularly simple, their space-time dynamical ones are easy to obtain. For example, in the two-dimensional Minkowski case, if the grid trajectories of the two satellites are non parallel straight lines, one can show that necessarily the satellites follow convergent geodesics, meanwhile if the grid trajectories are parallel straight lines, the two satellites are necessarily uniformly accelerated.

When the grid trajectories are arbitrary, an important general result may be prove: the knowledge of the dynamics of *one of the satellites* during a *relative acausal interval*, allows to know the dynamics of both of the satellites as well as that of the user at any later instant.

In other words: if 1 and 2 denote the two clocks, the relative acausal interval of the satellite 1 with respect to the satellite 2 at the "instant" τ_2 is the interval of the trajectory of the satellite 1 between its proper times $\tau'_1 = \tau_{21}$ and τ''_1 such that $\tau_{12} = \tau_2$. Then our result states that the dynamical knowledge of the satellite 1 during its proper time interval (τ'_1, τ''_1) suffices to know its dynamical trajectory for any $\tau_1 \geq \tau''_1$ and of 2 and the user for any $\tau_2 \geq \tau_{12}$.

The relative acausal interval of, say, the satellite 1 with respect to the satellite 2, at the "instant" τ_2 is the interval of the trajectory of the satellite 1 between its proper times $\tau'_1 = \tau_{21}$ and τ''_1 such that $\tau_{12} = \tau_2$. Then, in other words, our result states that the dynamical knowledge of the satellite 1 during its proper time interval (τ'_1, τ''_1) suffices to know its dynamical trajectory for any $\tau_1 \geq \tau''_1$ and of 2 and the user for any $\tau_2 \geq \tau_{12}$.

Of course, around the Earth more than four satellites will be convenient. Every four neighboring ones will constitute a *local chart* of the *atlas* of covariantly null coordinate systems enveloping the Earth. From this atlas, appropriate virtual global conventional coordinate system may be defined.

5 References

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